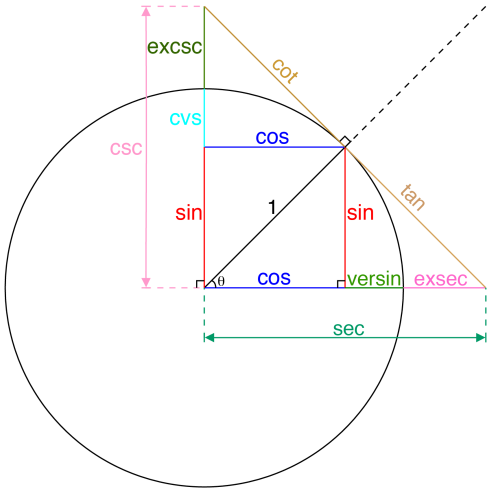
3 Trigonometry

3.1 Unit Circle



3.2 Domain and Range

- $\sin : \mathbb{R} \longrightarrow [-1, 1]$
- $\cos : \mathbb{R} \longrightarrow [-1, 1]$
- $\tan : \left\{x \in \mathbb{R} \mid x \neq \frac{\pi}{2} + k\pi\right\} \longrightarrow \mathbb{R}$
- $\cot : \{x \in \mathbb{R} \mid x \neq k\pi\} \longrightarrow \mathbb{R}$
- $\csc : \{x \in \mathbb{R} \mid x \neq k\pi\} \longrightarrow \mathbb{R} \setminus (-1, 1)$
- $\sec : \left\{x \in \mathbb{R} \mid x \neq \frac{\pi}{2} + k\pi\right\} \longrightarrow \mathbb{R} \setminus (-1, 1)$
- $\sin^{-1} : [-1, 1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
- $\cos^{-1} : [-1, 1] \longrightarrow [0, \pi]$
- $\tan^{-1} : \mathbb{R} \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

3.3 Pythagorean Identities

(i) $\sin^2(x) + \cos^2(x) = 1$
(ii) $\tan^2(x) + 1 = \sec^2(x)$
(iii) $1 + \cot^2(x) = \csc^2(x)$

3.4 Periodicity Identities

(i) $\sin(x \pm 2\pi) = \sin(x)$
(ii) $\cos(x \pm 2\pi) = \cos(x)$
(iii) $\tan(x \pm \pi) = \tan(x)$
(iv) $\cot(x \pm \pi) = \cot(x)$
(v) $\csc(x \pm 2\pi) = \csc(x)$
(vi) $\sec(x \pm 2\pi) = \sec(x)$

3.5 Reciprocal Identities

(i) $\cot(x) = \frac{1}{\tan(x)}$
(ii) $\csc(x) = \frac{1}{\sin(x)}$
(iii) $\sec(x) = \frac{1}{\cos(x)}$

3.6 Quotient Identities

(i) $\tan(x) = \frac{\sin(x)}{\cos(x)}$
(ii) $\cot(x) = \frac{\cos(x)}{\sin(x)}$

3.7 Sum Identities

(i) $\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
(ii) $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
(iii) $\tan(x + y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$

3.8 Difference Identities

(i) $\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
(ii) $\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
(iii) $\tan(x - y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

3.9 Double Angle Identities

(i) $\sin(2x) = 2\sin(x)\cos(x)$
(ii) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(iii) $\cos(2x) = 2\cos^2(x) - 1 \Rightarrow \cos^2(x) = \frac{\cos(2x)+1}{2}$
(iv) $\cos(2x) = 1 - 2\sin^2(x) \Rightarrow \sin^2(x) = \frac{1-\cos(2x)}{2}$
(v) $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

3.10 Co-Function Identities

(i) $\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$
(ii) $\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$
(iii) $\tan\left(\frac{\pi}{2} - x\right) = \cot(x)$
(iv) $\cot\left(\frac{\pi}{2} - x\right) = \tan(x)$
(v) $\csc\left(\frac{\pi}{2} - x\right) = \sec(x)$
(vi) $\sec\left(\frac{\pi}{2} - x\right) = \csc(x)$

3.11 Even-Odd Identities

(i) $\sin(-x) = -\sin(x)$
(ii) $\cos(-x) = \cos(x)$
(iii) $\tan(-x) = -\tan(x)$
(iv) $\cot(-x) = -\cot(x)$
(v) $\csc(-x) = -\csc(x)$
(vi) $\sec(-x) = \sec(x)$

3.12 Half-Angle Identities

(i) $\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos(x)}{2}}$
(ii) $\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos(x)}{2}}$
(iii) $\tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos(x)}{2}}$
(iv) $\tan\left(\frac{x}{2}\right) = \frac{1-\cos(x)}{\sin(x)}$
(v) $\tan\left(\frac{x}{2}\right) = \frac{\sin(x)}{1+\cos(x)}$

3.13 Sum-to-Product Formulas

(i) $\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$
(ii) $\sin(x) - \sin(y) = 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$
(iii) $\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$
(iv) $\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

3.14 Product-to-Sum Formulas

(i) $\sin(x)\sin(y) = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$
(ii) $\cos(x)\cos(y) = \frac{1}{2}[\cos(x - y) + \cos(x + y)]$
(iii) $\sin(x)\cos(y) = \frac{1}{2}[\sin(x + y) + \sin(x - y)]$
(iv) $\cos(x)\sin(y) = \frac{1}{2}[\sin(x + y) - \sin(x - y)]$

3.15 Tangent expression

If $u = \tan\left(\frac{x}{2}\right)$: $\left[dx = \frac{2}{1+u^2}du\right]$

(i) $\cos(x) = \frac{1-u^2}{1+u^2}$
(ii) $\sin(x) = \frac{2u}{1+u^2}$
(iii) $\tan(x) = \frac{2u}{1-u^2}$

3.16 Hyperbolic Functions

(i) $\sinh(x) = \frac{e^x - e^{-x}}{2}$
(ii) $\cosh(x) = \frac{e^x + e^{-x}}{2}$
(iii) $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

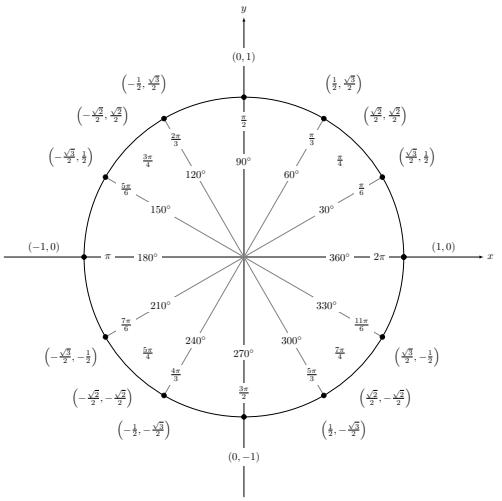
3.17 Laws of Sines

(i) $\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$

3.18 Laws of Cosines

(i) $a^2 = b^2 + c^2 - 2bc\cos(\alpha)$
(ii) $b^2 = a^2 + c^2 - 2ac\cos(\beta)$
(iii) $c^2 = a^2 + b^2 - 2ab\cos(\gamma)$

3.19 Degrees



θ		sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
Rad	Deg						
0 / 2π	0	0	1	0	Undef	1	Undef
π/6	30	1/2	√3/2	√3/3	2	2√3/3	√3
π/4	45	√2/2	√2/2	1	√2	√2	1
π/3	60	√3/2	1/2	√3	2√3/3	2	√3/3
π/2	90	1	0	Undef	1	Undef	0
2π/3	120	√3/2	− 1/2	− √3	2√3/3	-2	− √3/3
3π/4	135	√2/2	− √2/2	-1	√2	− √2	-1
5π/6	150	1/2	− √3/2	− √3/3	2	− 2√3/3	− √3
π	180	0	-1	0	Undef	-1	Undef
7π/6	210	− 1/2	− √3/2	√3/3	-2	− 2√3/3	√3
5π/4	225	− √2/2	− √2/2	1	− √2	− √2	1
4π/3	240	− √3/2	− 1/2	√3	− 2√3/3	-2	√3/3
3π/2	270	-1	0	Undef	-1	Undef	0
5π/3	300	− √3/2	1/2	− √3	− 2√3/3	2	− √3/3
7π/4	315	− √2/2	√2/2	-1	− √2	√2	-1
11π/6	330	− 1/2	√3/2	− √3/3	-2	2√3/3	− √3