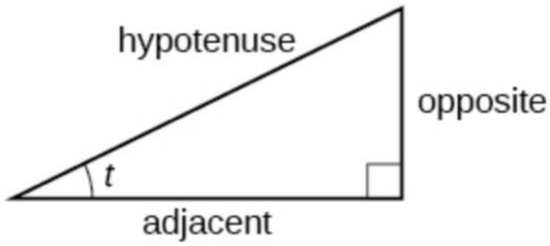
7.2 – Right Triangle Trigonometry

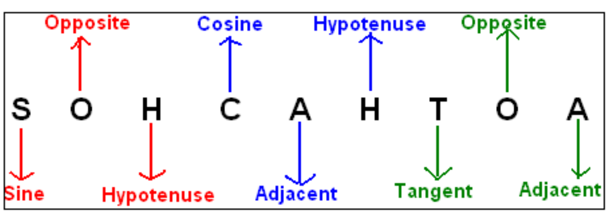
Learning Objectives

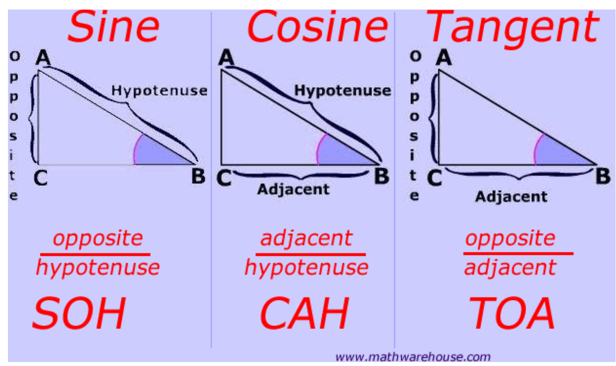
In this section you will:

- Use right triangles to evaluate trigonometric functions.
- Use the definitions of trigonometric functions of any angle.
- Use right-triangle trigonometry to solve applied problems.



The **adjacent side** is the side closest to the angle, *x*. (Adjacent means "next to.")
The **opposite side** is the side across from the angle, *y*.
The **hypotenuse** is the side of the triangle opposite the right angle, 1.





How To:

Given the side lengths of a right triangle and one of the acute angles, find the sine, cosine, and tangent of that angle.

1. Find the sine as the ratio of the opposite side to the hypotenuse.
2. Find the cosine as the ratio of the adjacent side to the hypotenuse.
3. Find the tangent as the ratio of the opposite side to the adjacent side.

Example Evaluating a Trigonometric Function of a Right Triangle

Given the triangle shown in Figure 3, find the value of tan α.

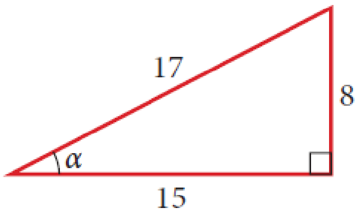
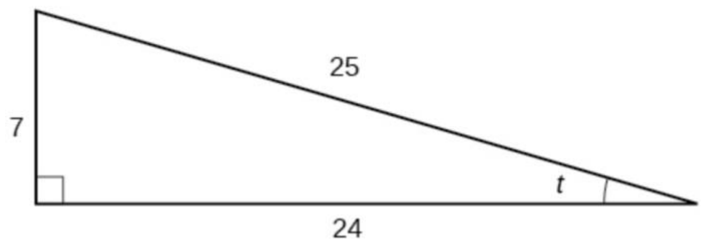


Figure 3

Ans: $\tan \alpha = \frac{opposite}{adjacent} = \frac{8}{15}$

Example: Given the triangle, find the value of $\sin t$, $\cos t$, & $\tan t$.



Ans:

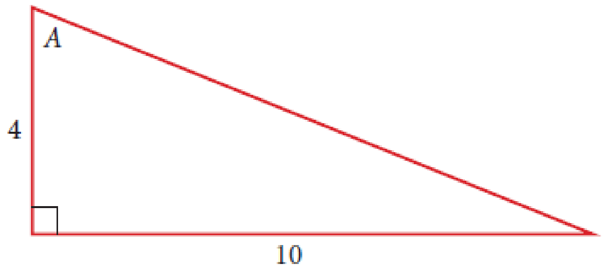
Sin $t = \frac{opposite}{hypotenuse} = \frac{7}{25}$

Cos $t = \frac{adjacent}{hypotenuse} = \frac{24}{25}$

tan $t = \frac{opposite}{adjacent} = \frac{7}{24}$

Example Evaluating a Trigonometric Function of a Right Triangle

Given the triangle shown below, find the value of cos *A*.



Ans: :

Lets first find the hypotenuse first, so we use Pythagoras theorem to find it. Let c be the hypotenuse. Then:

$4^2 + 10^2 = c^2$

$16+100 = c^2$

$116 = c^2$

$\sqrt{116} = c$

$2\sqrt{29} = c$

: $\text{Cos } \alpha = \frac{adjacent}{hypotenuse} = \frac{4}{2\sqrt{29}} = \frac{2}{\sqrt{29}}$

Reciprocal Functions

In addition to sine, cosine, and tangent, there are three more functions. These functions are the reciprocals of the first three functions.

Secant	$\sec t =$	$\sin t = \frac{1}{\csc t}$	$\csc t = \frac{1}{\sin t}$
Cosecant	$\csc t =$	$\cos t = \frac{1}{\sec t}$	$\sec t = \frac{1}{\cos t}$
Cotangent	$\cot t =$	$\tan t = \frac{1}{\cot t}$	$cot\ t = \frac{1}{\tan t}$

Many problems ask for all six trigonometric functions for a given angle in a triangle. A possible strategy to use is to find the sine, cosine, and tangent of the angles first. Then, find the other trigonometric functions easily using the reciprocals.

How To:

Given the side lengths of a right triangle, evaluate the six trigonometric functions of one of the acute angles.

1. If needed, draw the right triangle and label the angle provided.
2. Identify the angle, the adjacent side, the side opposite the angle, and the hypotenuse of the right triangle.
3. Find the required function:
   - sine as the ratio of the opposite side to the hypotenuse
   - cosine as the ratio of the adjacent side to the hypotenuse
   - tangent as the ratio of the opposite side to the adjacent side
   - secant as the ratio of the hypotenuse to the adjacent side
   - cosecant as the ratio of the hypotenuse to the opposite side
   - cotangent as the ratio of the adjacent side to the opposite side

Example **Evaluating Trigonometric Functions of Angles Not in Standard Position**
Using the triangle shown in **Figure 6**, evaluate sin α, cos α, tan α, sec α, csc α, and cot α.

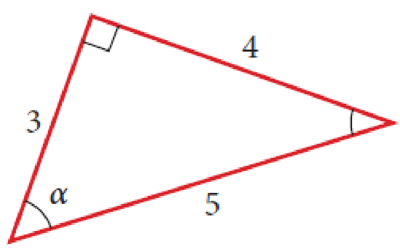


Figure 6

Ans:

Sin $\alpha = \frac{opposite}{hypotenuse} = \frac{4}{5}$

Csc $\alpha = \frac{hyotenuse}{opposite} = \frac{5}{4}$

Cos $\alpha = \frac{adjacent}{hypotenuse} = \frac{3}{5}$

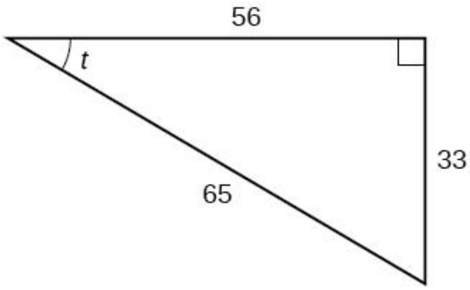
Sec $\alpha = \frac{hyotenuse}{adjacent} = \frac{5}{3}$

tan $\alpha = \frac{opposite}{adjacent} = \frac{4}{3}$

Cot $\alpha = \frac{adjacent}{opposite} = \frac{3}{4}$

Try It:

Using the triangle shown in Figure, evaluate sin t, cos t, tan t, sec t, csc t, and cot t.



Ans:

Sin α = $\frac{opposite}{hypotenuse} = \frac{33}{65}$

Csc α = $\frac{hyotenuse}{opposite} = \frac{65}{33}$

Cos α = $\frac{adjacent}{hypotenuse} = \frac{56}{65}$

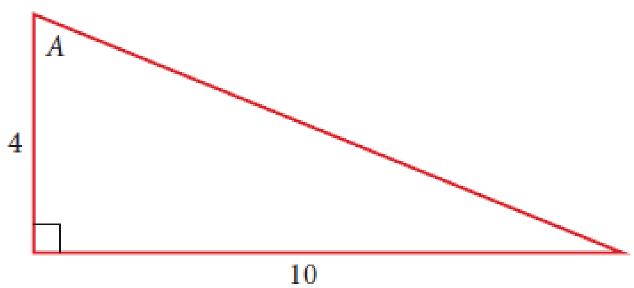
Sec α = $\frac{hyotenuse}{adjacent} = \frac{65}{56}$

tan α = $\frac{opposite}{adjacent} = \frac{33}{56}$

Cot α = $\frac{adjacent}{opposite} = \frac{56}{33}$

Example Evaluating a Trigonometric Function of a Right Triangle

Using the triangle shown below, find sin *A*, cos *A*, tan *A*, sec *A*, csc *A*, and cot *A*.



Ans: :

Lets first find the hypotenuse first, so we use Pythagoras theorem to find it. Let c be the hypotenuse. Then:

$4^2 + 10^2 = c^2$

$16+100 = c^2$

$116 = c^2$

$\sqrt{116}$ =c
$2\sqrt{29}$=c

Ans:

Sin $\alpha = \frac{opposite}{hypotenuse} = \frac{10}{2\sqrt{29}} = \frac{5}{\sqrt{29}}$

Csc $\alpha = \frac{hyotenuse}{opposite} = \frac{\sqrt{29}}{5}$

Cos $\alpha = \frac{adjacent}{hypotenuse} = \frac{4}{2\sqrt{29}} = \frac{2}{\sqrt{29}}$

Sec $\alpha = \frac{hyotenuse}{adjacent} = \frac{\sqrt{29}}{2}$

tan $\alpha = \frac{opposite}{adjacent} = \frac{5}{2}$

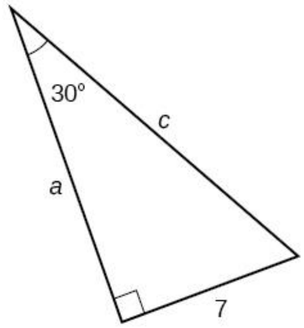
Cot $\alpha = \frac{adjacent}{opposite} = \frac{2}{5}$

Using Trigonometric Functions
How To:
Given a right triangle, the length of one side, and the measure of one acute angle, find the remaining sides.

1. For each side, select the trigonometric function that has the unknown side as either the numerator or the denominator. The known side will in turn be the denominator or the numerator.
2. Write an equation setting the function value of the known angle equal to the ratio of the corresponding sides.
3. Using the value of the trigonometric function and the known side length, solve for the missing side length.

Be aware: To find the cosine and sine of angles other than the special angles, we turn to a computer or calculator.
Most calculators can be set into "degree" or "radian" mode, which tells the calculator the units for the input value.

Example Finding Missing Side Lengths Using Trigonometric Ratios
Find the unknown sides of the triangle in Figure 11.



Ans:

Here we are suppose to find a and c.

Sin *30*° = $\frac{opposite}{hypotenuse} = \frac{7}{\text{c}}$

$$\frac{1}{2} = \frac{7}{c}$$

Cross multiply, to get

C=14.

Now you can either use Pythagoras's theorem or trig functions to compute a.

Cos α = $\frac{adjacent}{hypotenuse} = \frac{a}{14}$

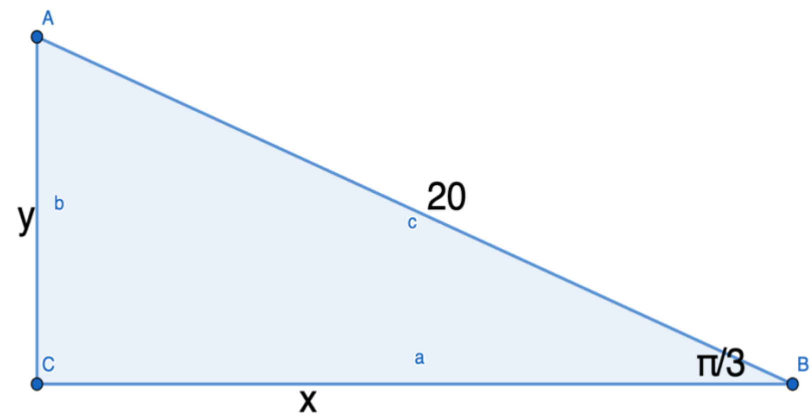
$$\frac{\sqrt{3}}{2} = \frac{a}{14}$$

Cross-multiply to get;

$$14\frac{\sqrt{3}}{2} = a$$

$$\frac{7\sqrt{3}}{2} = a$$

Try It: A right triangle has one angle of $\frac{\pi}{3}$ and a hypotenuse of 20. Find the unknown sides and angle of the triangle.



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Ans:

Here we are suppose to find adjacent and opposite side.

Sin$\frac{\pi}{3}$ = $\frac{opposite}{hypotenuse} = \frac{x}{20}$

$$\frac{\sqrt{3}}{2} = \frac{y}{20}$$

Cross multiply, to get

$$\frac{20\sqrt{3}}{2} = y$$

$$10\sqrt{3} = y$$

Now you can either use Pythagoras's theorem or trig functions to compute a.

Cos$\frac{\pi}{3}$ = $\frac{adjacent}{hypotenuse} = \frac{x}{20}$

$$\frac{1}{2} = \frac{x}{20}$$

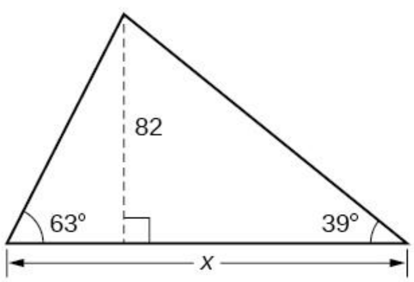
Cross-multiply to get;

$$\frac{20}{2} = x$$

10=x

Example Finding Missing Side Lengths Using Trigonometric Ratios

Find *x*.



a	b

tan *63*°. = $\frac{opposite}{adjacent} = \frac{82}{\mathrm{a}}$

$$\tan 63° = \frac{82}{\mathrm{a}}$$

Cross-multiply to get,

a= $\frac{82}{\tan 63°}$ = 41.7811

Similarly,

Tan 39 ° = $\frac{opposite}{adjacent} = \frac{82}{\mathrm{b}}$

$\text{Tan } 39° = \frac{82}{b}$

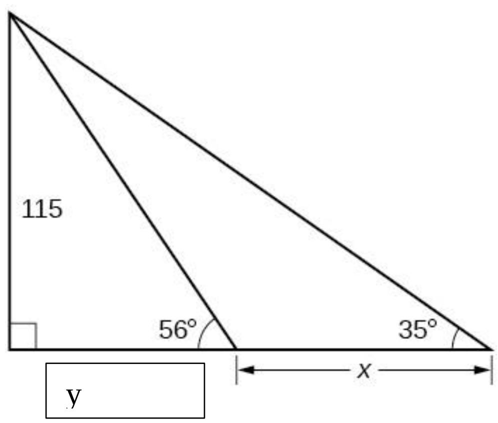
Cross-multiply to get,

b= $\frac{82}{\text{Tan } 39°}$ = 101.262

x=a+b= 41.7811+101.262= 143.043

Example Finding Missing Side Lengths Using Trigonometric Ratios

Find *x*.



Ans:

There are two ways to do this problem. Using trigonometry or using the property of similar triangles.

$\text{Tan}56° = \frac{opposite}{adjacent} = \frac{115}{y}$

$\text{Tan}56° = \frac{115}{y}$

Cross multiply to get;

$y = \frac{115}{\text{Tan}56°} = 77.5685$

$\text{Tan}35°. = \frac{opposite}{adjacent} = \frac{115}{x+y}$

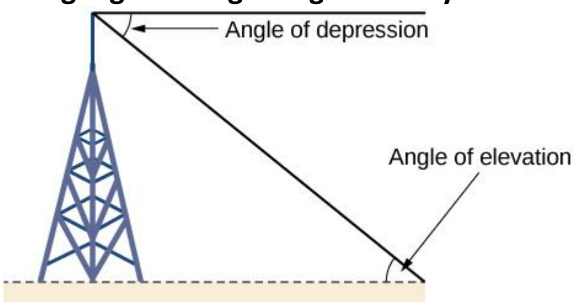
$0.7=\frac{115}{x+77.5685}$

Cross multiply to get;

$x+77.5685=\frac{115}{0.7}=164.286$

x=164.286-77.5685=86.72

Using Right Triangle Trigonometry to Solve Applied Problems



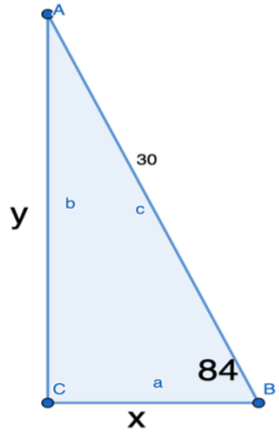
The angle of **angle of elevation** of an object above an observer relative to the observer is the angle between the horizontal and the line from the object to the observer's eye. The angle of **angle of depression** of an object below an observer relative to the observer is the angle between the horizontal and the line from the object to the observer's eye.

These right triangles created have sides that represent the unknown height, the measured distance from the base, and the angled line of sight from the ground to the top of the object. Knowing the measured distance to the base of the object and the angle of the line of sight, we can use trigonometric functions to calculate the unknown height.

How To:
Given a tall object, measure its height indirectly.

1. Make a sketch of the problem situation to keep track of known and unknown information.
2. Lay out a measured distance from the base of the object to a point where the top of the object is clearly visible.
3. At the other end of the measured distance, look up to the top of the object. Measure the angle the line of sight makes with the horizontal.
4. Write an equation relating the unknown height, the measured distance, and the tangent of the angle of the line of sight.
5. Solve the equation for the unknown height.

Example Measuring a Height Indirectly
A 30-ft ladder leans against a building so that the angle between the ground and the ladder is 84°. How high does the ladder reach on the building?



Ans:

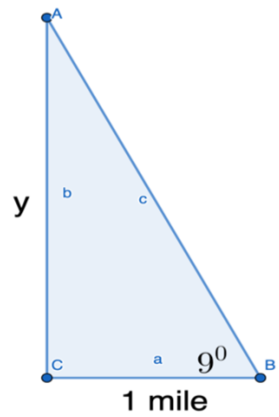
We only need y for this questions, that is the height for this building. We have hypotenuse 30 and angle 84° is given so we need Sine function to compute y.

Sin 84° $= \frac{opposite}{hypotenuse} = \frac{y}{30}$

0.9945 = $\frac{y}{30}$

Y=0.9945*30=29.84 feet

The angle of elevation to top of a Building in New York is found to be 9 degrees from the ground at a distance of 1 mile from the base of the building. Using this information, find the height of the building. Round to the tenths. Hint 1 mile=5280 feet.



Ans:

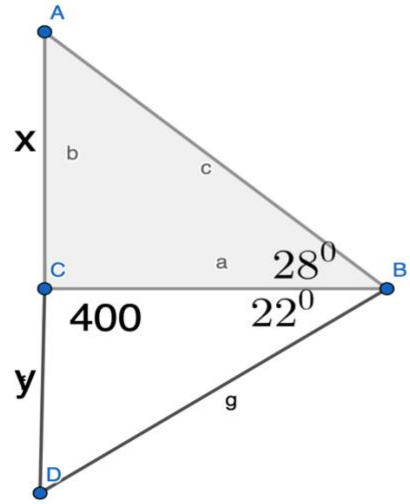
We only need y for this questions, that is the height for this building. We have adjacent side 1mile and angle 9° is given so we need tan function to compute y.

Tan 9° $= \frac{opposite}{hypotenuse} = \frac{y}{1}$

Tan 9° *= y (in miles)*

Y= Tan 9°*5280 =836.27 feet

A radio tower is located 400 feet from a building. From window in the building, a person determines that the angle of elevation to the top of the tower is 28° and the angle of depression to the bottom of the tower is 22°. How tall is the tower?



Assume B is the position of the guy looking from Window, 28° is angle of eleveation and 22° is angle of depression.. The height of the tower will be x+y. The opposite side is given for both of the traingales and 400 is common adjacent side, so we use tangentfunction.

Tan 28 ° = $\frac{opposite}{hypotenuse} = \frac{x}{400}$

$\text{Tan } 28° = \frac{x}{400}$

Cross multiply to get;

x= $400 \text{ Tan } 28° = 212.68\ feet$

Again, lets find y

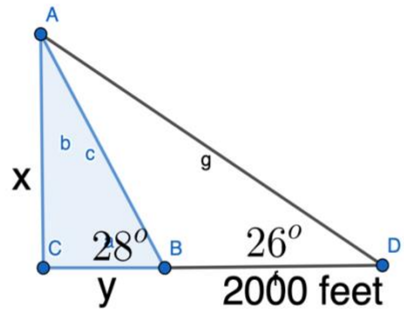
Tan 22° = $= \frac{y}{400}$

Cross multiply to get;

y= $400 \text{ Tan } 22° = 161.61$

Height of the building is x+y=212.68+161.61=374.29 feet

A survey team is trying to estimate the height of a mountain above a level pain. From one point on the plain, they observe that the angle of elevation to the top of the mountain is 26°. From a point 2000 feet closer to the mountain along the plain, they find that the angle of elevation is 28°. How high (in feet) is the mountain?



Assume that survey team starts at point D. They measure an angle of 26 degree. Now, they move 2000 feet closer and measure an angle of 28 degree. Our goal is to find the height of mountain x. The problem here is we do not know y.

Tan 28 ° = $\frac{opposite}{hypotenuse} = \frac{x}{\mathrm{y}}$

Tan 28 ° $= \frac{x}{\mathrm{y}}$

0.5317=$\frac{x}{\mathrm{y}}$

0.5317y=x

y=$\frac{x}{0.5317}$ =1.8807x

y=1.8807x

The reason I separate y here is I want to use this later to replace y in terms of x so I can find height "x".

Tan 26 ° = $\frac{opposite}{hypotenuse} = \frac{x}{\text{y+2000}}$

Tan 26 ° $= \frac{x}{\text{2000+y}}$

0.4877=$\frac{x}{\text{y+2000}}$

0.4877(y+2000)=x

0.4877y+0.4877*2000=x

0.4877 y+975.4652=x

0.4877(1.8807x)+975.4652=x

0.9172x+975.4652=x

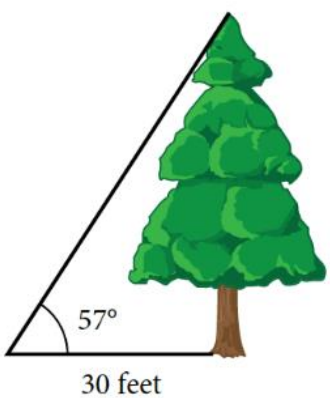
975.4652=x-0.9172x

975.4652=0.0828x

$$\frac{975.4651}{0.0828} = 1178.10\ feet = x$$

Practice Probelms

To find the height of a tree, a person walks to a point 30 feet from the base of the tree. She measures an angle of 57° between a line of sight to the top of the tree and the ground, as shown in Figure 13. Find the height of the tree.



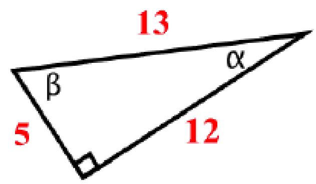
Try It: How long a ladder is needed to reach a windowsill 50 feet above the ground if the ladder rests against the building making an angle of $\frac{5\pi}{12}$ with the ground? Round to the nearest foot.

Example Measuring a Distance Indirectly

A 400-foot tall monument is located in the distance. From a window in a building, a person determines that the angle of elevation to the top of the monument is 18°, and that the angle of depression to the bottom of the tower is 3°. How far is the person from the monument?

Find the value of the trig functions indicated.

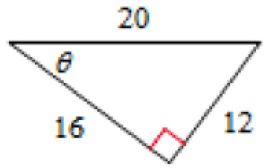
1.



$\sin\alpha =$

$\tan\beta =$

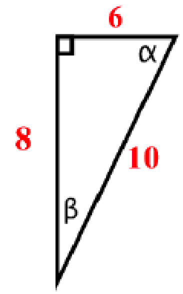
2.



$\cos\theta =$

$\tan\theta =$

3.



$\sin\alpha =$

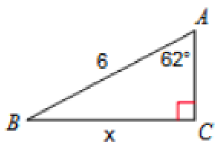
$\sin\beta =$

$\tan\alpha =$

$\cos\beta =$

Find the measure of the indicated side. Round to the nearest hundredth.

4.



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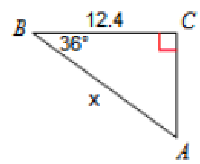
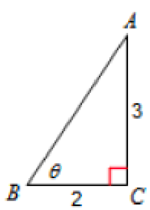


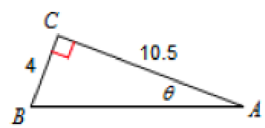
6. Given ΔDEF where $\angle D$ is a right angle. Find f if $m\angle E = 16°$ and $e = 4$. (Draw a picture!)

Find the measure of the indicated angle. Round to the nearest hundredth.

7.



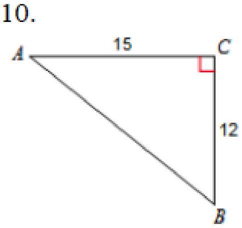
8.



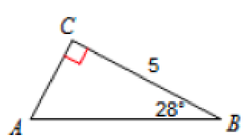
9. Given ΔDEF where $\angle D$ is a right angle. Find $m\angle E$ if $e = 11.7$ and $f = 8$. (Draw a picture!)

Solve each triangle. Round to the nearest hundredth.

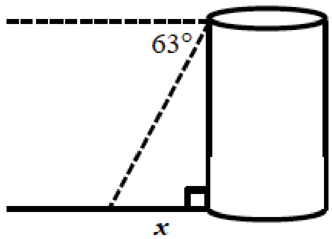
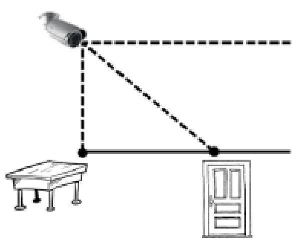
10.



11.



12. Given ΔDEF where $\angle D$ is a right angle and $m\angle F = 35°$ and $d = 16$. (Draw a picture!)

Label the picture given and then solve it. If no picture is given, draw your own and solve!	
13. The angle of depression is measured from the top of a 43 ft tower to a reference point on the ground. Its value is found to be $63°$. How far is the base of the tower from the point on the ground?  	14. The entrance of the old town library is 2.3 ft above ground level. A ramp from the ground level to the library entrance is scheduled to be built. The angle of elevation form the base of the ramp to its top is to be $15°$. Find the length of the ramp
15. A closed circuit TV camera is mounted on a wall 7.4 ft above a security desk in an office building. It is used to view an entrance door 9.3 ft from the desk. Find the angle of depression from the camera lens to the entrance door. 	16. A jet took off at a rate of 260 ft/s and climbed in a straight path for 3.2 min. What was the angle of ele vation of its path if its final altitude was 12,000 ft?
17. The angle of elevation from the bottom of the world's largest slide located in Peru, Vermont, is approximately $10.3°$. The slide has a vertical drop of 821 ft. Find the length of the slide.	18. The extension ladder on top of a 6 ft high hook and ladder truck is 150 ft long. If the angle of elevation of the ladder is $70°$, to what height on a building will the ladder reach?