River Parishes Community College

Math 1100: College Algebra

Quadratic Functions

5.1 Quadratic Functions and Parabolas

Semester
Fall/Spring Year—-

Department
Physical Science: Math

June 3, 2021

Learning Objectives

In this section, you will learn:
- ♣ Recognize characteristics of parabolas
- ♣ Undertstand how the graph of a parabola is related to its quadratic function
- ♣ Determine minimum or maximum value (turning points) in a quadratic function's
- ♣ Solve problems involving a quadratic function's minimum or maximum value

1 Quadratic Equation

A quadratic equation is a polynomial equation with degree two. In other words, it is an equation of the form

$$ax^2 + bx + c = 0,$$

where a, b and c are real numbers and $a \neq 0$ .The graph of a quadratic function is a U-shaped graph and is called **Parabolas**.

Examples of Quadratic equation

a. $x^2 - 1 = 0$
b. $3x^2 + 5x + 2 = 10$
c. $x^2 = 4$
d. $\frac{3}{2}x^2 + 7x = 5$

Examples of Non-Quadratic eunction

a. $f(x, y) = 3x + 2y$ function of two Variables
b. $0 = 3 + 2x$ The highest exponent (degree/power) is not 2
c. $0 = \sqrt{x} + 2$ radical (fractional) exponents.
d. $0 = \frac{2}{x} + 3$ Variable in denominator
e. $x^3 + 3x^2 + 9 = 0$ Third degree equation

2 Quadratic Forms

1. Standard (General) From

$$y = ax^2 + bx + c$$

eg

$$y = 2x^2 + 4x - 6$$

2. Vertex Form

$$y = a(x - h)^2 + k$$

eg

$$y = 2(x + 1)^2 - 8$$

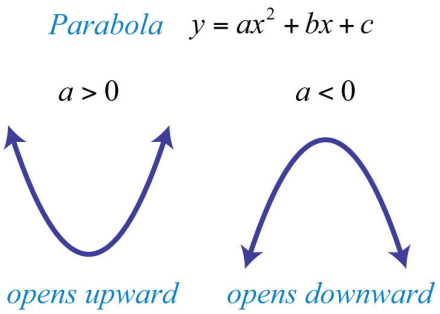
3. Factor Form

$$y = a(x - p)(x - q)$$

eg

$$y = 2(x + 3)(x - 1)$$

If $a > 0$, i.e a is positive, parabola opens up.
if $a < 0$, i.e a is negative, parabola opens down.



3 Key Features of Quadratic function

1. Vertex
   All quadratic have a **minimum** or **maximum** point which is also the **turning point** of the parabola. It is called the **vertex** of the parabola. The coordinates can be found using the following formulas:

$$x = -\frac{b}{2a}, \text{ and } \textbf{Vertex} = \left(-\frac{b}{2a}, f(-\frac{b}{2a}) \right) \text{ for } y = ax^2 + bx + c$$

$$\textbf{Vertex} = (h, k) \text{ for } y = a(x - h)^2 + k$$

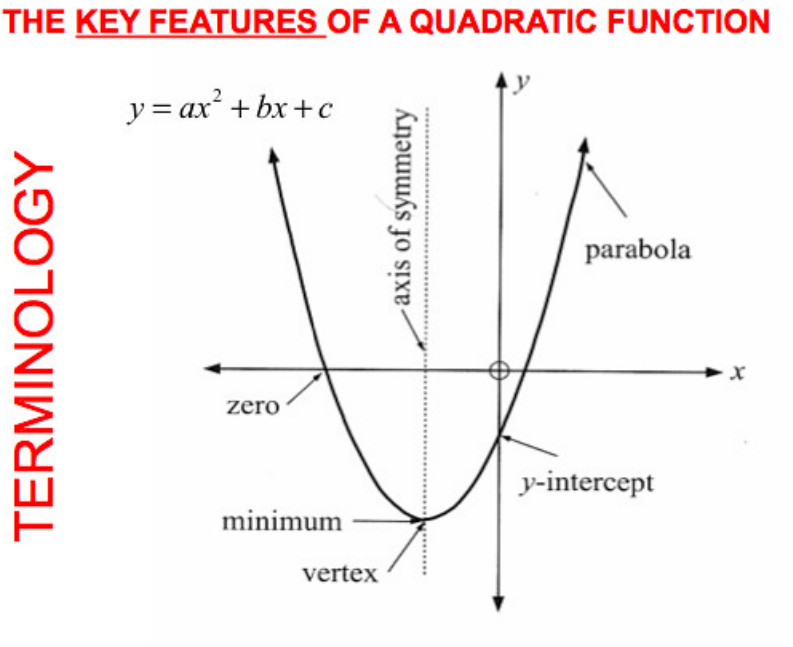
2. Axis of Symmetry
   Every quadratic is symmetrical with respect to some vertical line called **Axis of Symmetry**. It is a line that passes through the vertex, so the equation of line of symmetry is given by:

$$x = -\frac{b}{2a}, \text{ for } y = ax^2 + bx + c$$

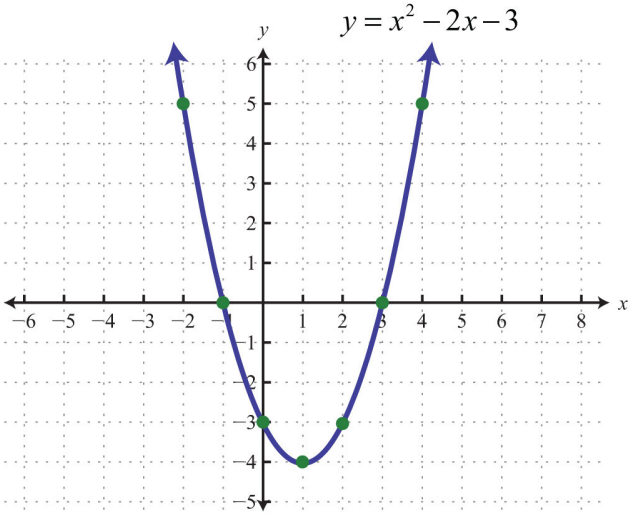
$$x = h, \text{ for } y = a(x - h)^2 + k$$

3. Y-intercept
   A quadratic graph always cross the y-axis at a point given by co-ordinates $(0, f(0))$. The x-coordinate is zero and y co-ordinate can be found by plugging $x = 0$ in quadratic equation

4. X-intercepts : roots, Zeros, Solutions
   A quadratic equation may or may not touch x-axis. If it touches x-axis, it may touch x-axis at two points or only one points. These points are called x-intercepts. They can be found by solving quadratic equations.



Examples:



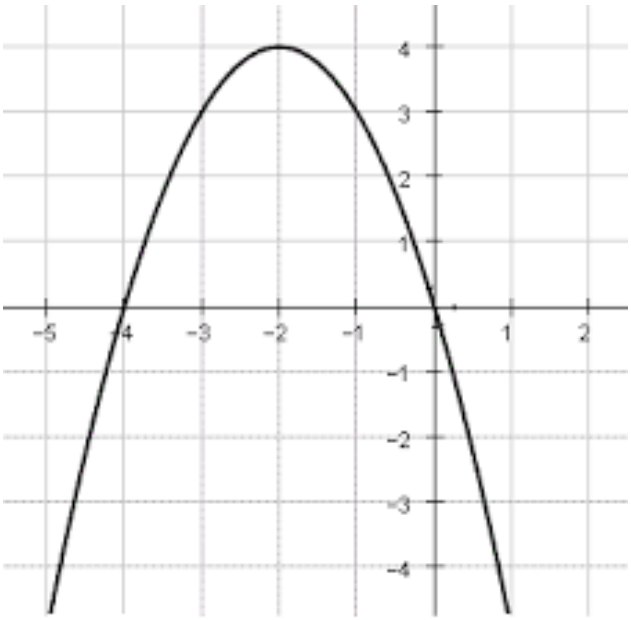
Fill the following

Vertex _____

Axis of Symmetry _____

Y-intercept _____

X-intercepts _____



Fill the following

Vertex _____

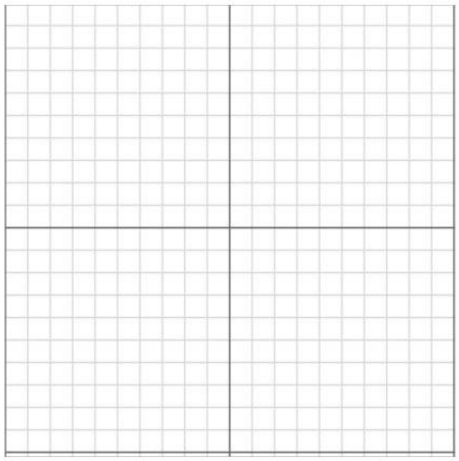
Axis of Symmetry _____

Y-intercept _____

X-intercepts _____

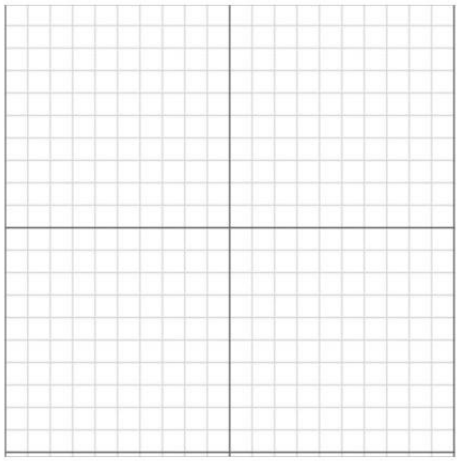
4 Graphing in Standard form: $y = ax^2 + bx + c$

Example Graph $f(x) = y = 2x^2 - 2x - 4$
To find axis of symmetry:
$x = -\frac{b}{2a}$ = _____ = _____
To find vertex, plug back _____ into _____
$f($_____$) = 2($_____$)^2 - 2$_____ $- 4$



Key features
a = _____, b = _____, c = _____
The graph opens UP _____ or Down _____
The graph has max _____ or min _____
Vertex _____
Axis of Symmetry _____
Y-intercept _____
X-intercepts _____
One point in Parabola : (x = _____, y = _____)

Example Graph $f(x) = y = -3x^2 - 6x + 1$
To find axis of symmetry:
$x = -\frac{b}{2a}$ = _____ = _____
To find vertex, plug back _____ into _____
$f($_____$) = -3($_____$)^2 - 6$_____ $+ 1$



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One point in Parabola : (x = _____, y = _____)

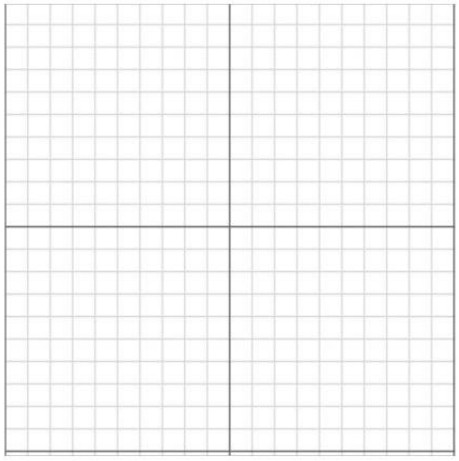
5 Graphing in Vertex form: $y = a(x-h)^2 + k$

In this form the vertex is given by (h, k).

Example Graph $f(x) = y = 2(x-3)^2 - 8$

To find axis of symmetry:
$x = h =$ _____

vertex: $(h, k) =$ _____



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The graph has max _____ or min _____
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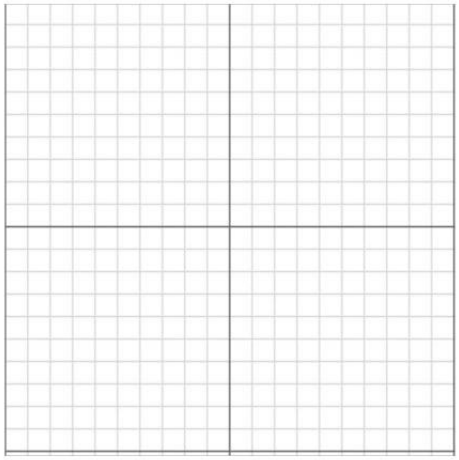
Axis of Symmetry _____
Y-intercept _____
X-intercepts _____
One point in Parabola : ($x =$ _____, $y =$ _____)

Example Graph $f(x) = y = -(x+3)^2 + 1$

To find axis of symmetry:
$x = h =$ _____

vertex: $(h, k) =$ _____



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The graph opens UP _____ or Down _____
The graph has max _____ or min _____
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Axis of Symmetry _____
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X-intercepts _____
One point in Parabola : ($x =$ _____, $y =$ _____)

6 Graphing in factored form: $y = a(x-p)(x-q)$

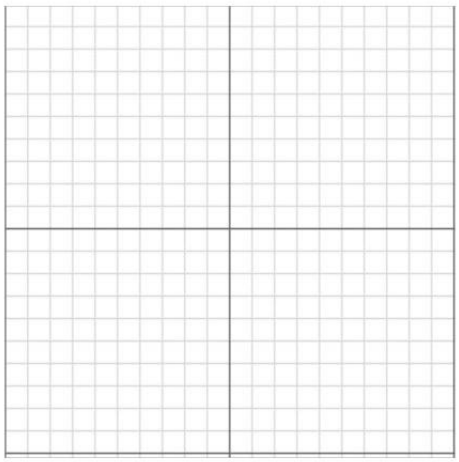
p and q are called _____ , _____, _____.
The axis of symmetry is given by formula

$$x = \frac{p+q}{2}.$$

Example Graph $f(x) = y = -2(x-3)(x-1)$

To find axis of symmetry:
$x = \frac{p+q}{2}$ = _____

To find vertex, plug back _____ into _____
$f($_____$) = -2($_____$ - 3)($_____$ - 1)$



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The graph opens UP _____ or Down _____
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Vertex _____
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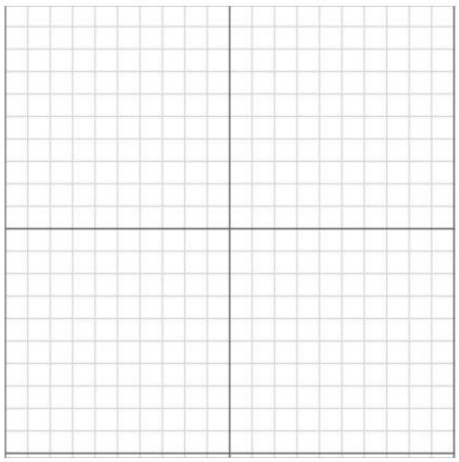
Y-intercept _____
X-intercepts _____
One point in Parabola : (x = _____, y = _____)

Example Graph $f(x) = y = (x+1)(x-1)$

To find axis of symmetry:
$x = \frac{p+q}{2}$ = _____

To find vertex, plug back _____ into _____
$f($_____$) = ($_____$ + 1)($_____$ - 1)$



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