River Parishes Community College

Math 1100: College Algebra

Linear Equations and Modeling

4.1 Introduction to Linear Functions

Semester
Sp/Fall Year

Department
Physical Science: Math

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Learning Objectives

In this section, you will learn:
♣ Distingusih the difference between Linear vs Non-linear Equations
♣ Interpret Linear Equations
♣ Learn the concept of Slope
♣ Given the equations of two lines, determine whether their graphs are parallel or perpendicular.
♣ Write the equation of a line parallel or perpendicular to a given line.

1 Linear Function

A linear function is an equation that can be written in the form

$$f(x) = y = mx + b$$

where m and b are real numbers (constants). Note that the power (exponent) of x is 1.

The graph of linear function is always a line. The linear function is popular in economics. It is attractive because it is simple and easy to handle mathematically. It has many important applications.

Examples of Linear Function

a. $y = 2x - 5$
b. $5x + 7y = 10$ can be rewritten as $y = -\frac{5x}{7} + \frac{10}{7}$
c. $f(x) = 2$

Examples of Non-linear Function

a. $f(x, y) = 3x + 2y$ function of two Variables
b. $y = x^2 + 2x$ The exponent (degree/power) is 2
c. $f(x) = \sqrt{x} + 2$ radical (fractional) exponents.
d. $f(x) = \frac{2}{x} + 3$ Variable in denominator

2 What is Slope?

Short answer: The slope between two points is rise over run. Rise is the difference of two y values and run is difference of two x values. What this means is that slope will tell how much higher the function moves (in y -direction) if you move over a certain amount to the right.

Consider the function $y = \frac{2}{1}x + 3 = 2x + 3$.
By using plotting technique we learned in earlier chapter we start at the point $(0, 3)$. All points that satisfy this equation fit on the line. So if we plug in $x = 1$, then $y = 5$. By continuing this process we end up with a graphed line.

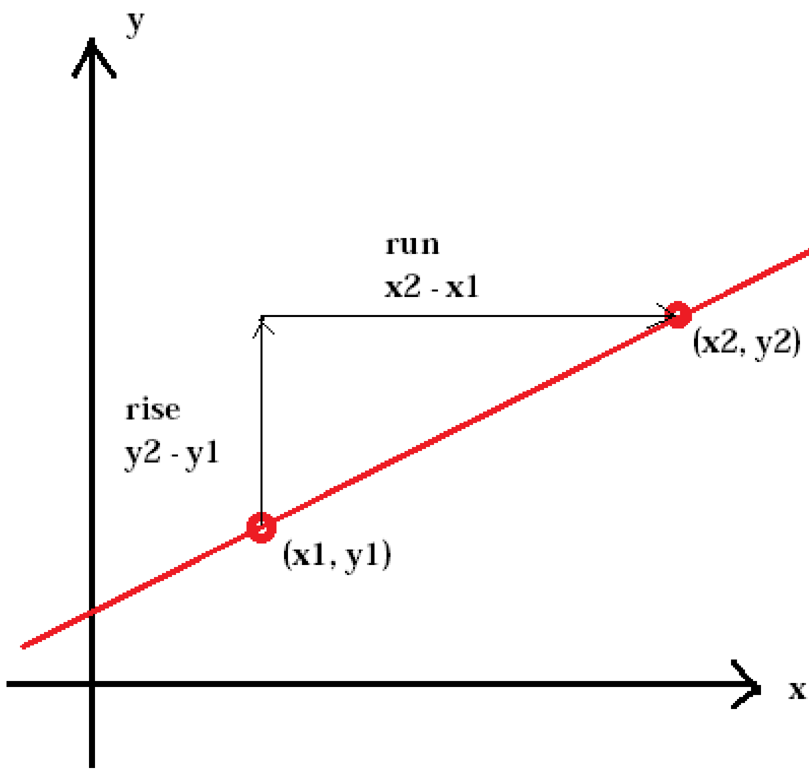
The slope of this line is the distance "up," (change in y) for every distance we go "right," (change in x).

So if we were to go 1 in the x direction (say from 0 to 1) we move up 2 in y direction and reach the point $(1,5)$. Similarly, the points $(1+1, 5+2) = (2,7)$ and $(2+1, 7+2) = (3,9)$ and $(3+1, 9+2) = (4,11)$ all fit the line.

In mathematical language, the slope m of the line is

Slope Equation

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$



The slope of line measures the steepness of a line. A line is either increasing, decreasing, horizontal or vertical.

Slope and Steepness

♣ A line is increasing if it goes up from left to right. The slope is positive, i.e. $m > 0$.
♣ A line is decreasing if it goes down from left to right. The slope is negative, i.e. $m < 0$.
♣ If a line is horizontal the slope is zero. This is a constant function.
♣ If a line is vertical the slope is undefined .

3 Slope-Intercept Form of a Line: $y = mx + b$

Let $(0, b)$ be the y-intercept of a line i.e it is the point where the line intersects the y axis. Let (x, y) be other any other point in the line. Now using the slope equation

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

$$m = \frac{y - b}{x - 0}$$

$$m = \frac{y - b}{x}$$

Multiplying both sides by x

$$mx = y - b$$

Adding b on both sides

$$mx + b = y$$

Besides being neat and simplified, slope-intercept form's advantage is that it gives two main features of the line it represents:

♣ **The slope is** m

♣ **The** y **intercept is** b**. i.e in order pairs :** $(0, b)$

Eg

The line $y = 2x + 1$ has slope 2 and y-intercept $(0, 1)$.

The fact that this form gives the slope and the y-intercept is the reason why it is called slope-intercept in the first place!

4 Characteristics of a line

A linear function is a function whose graph is a line. Linear functions can be written in the slope-intercept form of a line

$$f(x) = mx + b$$

where b is is the initial or starting value of the function (when input, $x = 0$), and m is the constant rate of change, or slope of the function. The y-intercept is at $(0, b)$.

5 Point-Slope Form of a Line

Let (x_1, y_1) be the given point of a line. Then, let (x, y) be any other point in the given line. Using the slope formula we

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

$$m = \frac{y - y_1}{x - x_1}$$

Multiplying both sides by $(x - x_1)$

$$m(x - x_1) = y - y_1$$

This form gives the slope and a point in the line, hence this is called point-slope Form of a line.

Eg 1: Find the Equation of a line that passes through point $(1, 5)$ and has slope -2

Write the slope intercept form:

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -2(x - 1)$$

Eg 2: Find the Equation of a line that passes through points $(1, 4)$ and $(6, 19)$

Step 1: Find the slope :

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

$$m = \frac{19 - 4}{6 - 1} = \frac{15}{5} = 3.$$

Step 2: Use the point slope form:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 3(x - 1)$$

or

$$y - 19 = 3(x - 6)$$

6 Standard Form of a Line

Another way that we can represent the equation of a line is in standard form. Standard form is given as

$$Ax + By = C$$

where A, B and C are integers. The x- and y-terms are on one side of the equal sign and the constant term is on the other side. This form is nice only because it does not involve any fractions. However, to find the slope and y-intercept, it needs to be converted to other forms.

Three Forms of the Equation of Line

1. Point Slope form

$$y = mx + b.$$

2. Slope Intercept form

$$y - y_1 = m(x - x_1).$$

3. Standard Form

$$Ax + By = C.$$

Three Scenarios for finding the Equation of Line

♣ Given a slope and y-intercept , find the equation of a line. (Eg in page 4)
♣ Given a slope and a point, find the equation of a line. (Eg 1 in page 5)
♣ Given two points in a line, find the equation of a line. (Eg 2 in page 5)

7 Horizontal Lines and Vertical Lines

Horizontal Line

All the points in the horizontal line have the same y-co-ordinates, so the equation of a horizontal line is given by

$$y = b.$$

The horizontal line does not rise at all, so the slope is zero .

Vertical Line

All the points in the vertical line have the same x-co-ordinates, so the equation of a horizontal line is given by

$$x = c.$$

The vertical line does run at all, so the slope is undefined because the denominator goes to zero in the slope formula .

Eg 1: Find the Equation of a line that passes through point $(1, 5)$ and $(1, -3)$

Since they have the same x co-ordiante, it is a vertical line. The slope is undefined and the equation of the line is

$$x = 1$$

.

Eg 2: Find the Equation of a line that passes through points $(1, 4)$ and $(-3, 4)$

Since they have the same y coordinate , it is a horizontal line . The slope is zero and the equation of the line is

$$y = 4.$$

8 Interpreting Linear Equations

Example 1

At $12:00$ P.M. the volume of water in a tank started changing steadily over time. The volume V measured in gallons, t minutes after the water volume started to change is given by the equation

$$V = 3200 - 1.6t$$

The equation tells us that the volume of water in the tank was initially 3200 gallons gallons, and that the volume is decreasing by 1.6 gallons per minutes.

Example 2

Paul is planning to sell bottled water at the local carnival. Paul's profit (in dollars) from selling b bottles of water is given by the formula

$$P(b) = 1.5b - 223$$

It means Paul's profit is increasing at a rate of 1.5 dollars per each bottle sold.

9 Parallel and Perpendicular Lines

Parallel Lines

The slopes of parallel lines are always equal.

♣ Example:

♠ The equations $y = 4x + 1$ and $y = 4x + 3$ are parallel because they both have a slope of 4.

Perpendicular lines

The slopes of perpendicular lines are always opposite reciprocals of each other.

♣ Example:

- The equations $y = \frac{2}{3}x+2$ and $y = -\frac{3}{2}x+5$ are perpendicular because their slopes are negative reciprocals of each other.
- The term opposite reciprocal simply means to flip the slope and change the sign.
  - For example, if $m = \frac{4}{5}$, then a slope perpendicular to that would be $m = -\frac{5}{4}$.

Two Slopes that are perpendicular will always have product of -1

- For example:
  $\frac{4}{5} * -\frac{5}{4} = -1$

3 steps to write Parallel and Perpendicular Lines

1. Determine the slope of the new line.

2. Pick which equation to use:

   - Slope-intercept: $y = mx + b$.

   - Point-Slope: $y - y_1 = m(x - x_1)$.

3. Substitute values.

Example: Write the equation of a line perpendicular to $y = -\frac{1}{3}x - 5$ through the point $(-1, 4)$.
Write the final answer in slope-intercept form

Step 1: **Determine the slope of the new line**.
Since it must be perpendicular to the given line, the slope of our new line must be $m = 3$.

Step 2: **Pick equation.**
Since we have the point $(-1, 4)$, we should use the point-slope equation of a line $y - y_1 = m(x - x_1)$.

Step 3: **Substitute values**.
Since $m = 3$ and $(x_1, y_1) = (-1, 4)$, we can substitute those into the point-slope equation:

$$y - y - 1 = m(x - x_1)$$

$$y - 4 = 3(x - (-1)) = 3(x + 1)$$

Now the problem says to write the final answer in slope-intercept form. To do this, we simply solve for y.

$$y - 4 = 3(x + 1)$$

$$y - 4 = 3x + 3$$

$$y = 3x + 7$$

So the line perpendicular to $y = -\frac{1}{3}x - 5$ through the point $(-1, 4)$ is $y = 3x + 7$