River Parishes Community College

Math 1100: College Algebra

Functions

3.3 Rates of Change and Behavior of functions

Semester
Sp/Fall Year

Department
Physical Science: Math

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Learning Objectives

In this section, you will learn:
♣ Find the average rate of change of a function.
♣ Use a graph to determine where a function is increasing, decreasing, or constant.
♣ Use a graph to locate maxima and local minima
♣ Use a graph to locate the absolute maximum and absolute minimum

1 Rate of Change

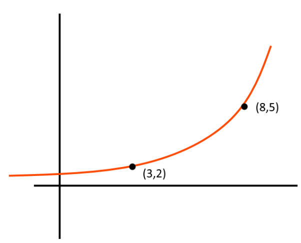
In simple terms , rate of change is the speed of your car. It will tell you how fast or slow you are driving. For example: If the speed of your car is 10 miles per hour, this means for every hours you drive , you go 10 more miles. In 2 hours, you will travel $10 + 10 = 20$ miles, in 3 hours $10 + 10 + 10 = 30$ miles and so on. Thus, Rate of change from one point to another point is actually the slope of a line passing through this two points.
Slope is the ratio of the vertical and horizontal changes between two points on a line.The vertical change between two points is called the rise, and the horizontal change is called the run. The slope equals the rise divided by the run.
To find the average rate of change of a function between any two points on its graph, calculate the slope of the line containing the two points $x = a$ and $x = b$.

$$\text{Average rate of change } = \frac{f(b)-f(a)}{b-a}$$

Example 1 Find the average rate of change of $f(x) = 4x^3 + 5$ from $x = 3$ to $x = 4$.
Solution:
$f(3) = 4(3)^3 + 5 = 113$
$f(5) = 4(5)^3 + 5 = 505$

Now rate of change is given by the slope $= \frac{f(5)-f(3)}{5-3} = \frac{505-113}{5-3} =$

Example 2 Use the graph below to find the average rate of change from $x = 3$ to $x = 8$.



Solution:
Note that $f(3) = 2$ and $f(8) = 5$.
Now rate of change is given by the slope $= \frac{f(8)-f(3)}{8-3} = \frac{5-2}{8-3} = \frac{3}{5}$

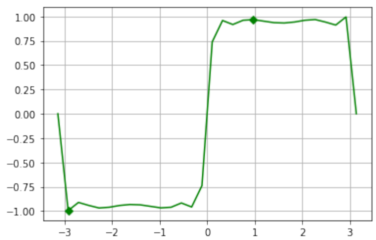
Example 3 The following chart shows "living wage" jobs in Rochester per 1000 working age adults over a 5 year period.

Year	1997	1998	1999	2000	2001
Jobs	675	730	775	815	845

What is the average rate of change in the number of living wage jobs from 1997 to 1999?
Solution:
Average rate of change is given by the slope = $\frac{y_2-y_1}{x_2-x_1} = \frac{775-675}{1999-1997} = \frac{100}{2} = 50$ jobs/year.

What is the average rate of change in the number of living wage jobs from 1999 to 2001?
Solution:
Average rate of change is given by the slope = $\frac{y_2-y_1}{x_2-x_1} = \frac{845-775}{2001-1999} = \frac{70}{2} = 35$ jobs/year.

Example 4 Find the rate of change from the following graph using two marked points.

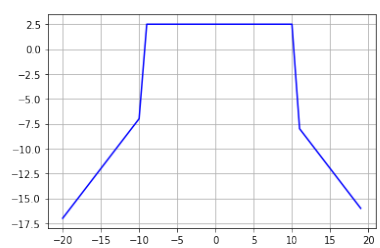


Solution:
The two points are marked at $x = 1$ and $x = -3$. The y-value at $x = 1$ is 1 and the y-value at $x = -3$ is -1.
Average rate of change is given by the slope = $\frac{y_2-y_1}{x_2-x_1} = \frac{-1-1}{-3-1} = \frac{-2}{-4} = \frac{1}{2}$.

2 Behavior of a function

If a function is decreasing, then as the values of x get bigger, the values of the function get smaller. The function falls as you look from left to right.
If a function is increasing, then as the values of x get bigger, the values of the function also get bigger. The function rises from left to right.
If a function is constant, then as the values of x get bigger, the values of the function remain unchanged. The function is a horizontal straight line.

Example 5 Find the interval of increasing, decreasing and constant from the following graph.



Solution:
The function first rises from left to right $(-\infty, -9)$, becomes constant on middle region $(-9, 10)$ and then again falls from left to right $(10, \infty)$.