River Parishes Community College

Math 1100: College Algebra

Equations and Inequalities

2.7 Linear Inequalities and Absolute Value Inequalities

Semester
Fall/Spring Year

Department
Physical Science: Math

June 3, 2021

Learning Objectives

In this section, you will learn:

♣ Solve equations involving rational exponents.
♣ Use Interval Notation
♣ Use Properties of Inequalities
♣ Solve Inequalities
♣ Solve Combined Inequalities
♣ Solve Inequalities Involving Absolute Value

Common Symbols:

$<, >$ this will corresponds to open holes or parenthesis.
$\leq, \geq$ will corresponds to closed holes or Brackets.

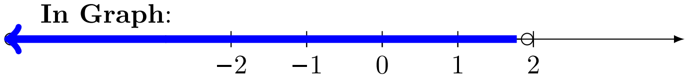
The inequality $x < a$ represents all the numbers less than a.

For example:

$x < 2$

Interval Notation: $(-\infty, 2)$.

In Graph:

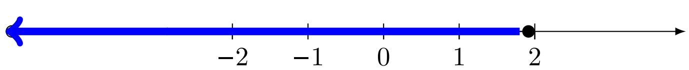


Similarly, the inequality $x \leq a$ represents all the numbers less than or equal to a.
For example:

$x \leq 2$

Interval Notation $(-\infty, 2]$

In Graph

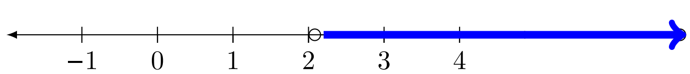


Likewise, The inequality $x > a$ represents all the numbers greater than a.

For example $x > 2$.

In Interval Notation: $(2, \infty)$.

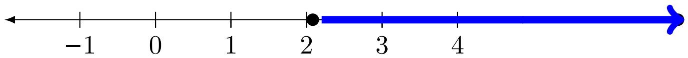
In Graph:



Similarly, the inequality $x \geq a$ represents all the numbers greater than or equal to a. It can also be written as $x \in [a, \infty)$ For example $x \geq 2$.

In Interval Notation: $[2, \infty)$.

In Graph:



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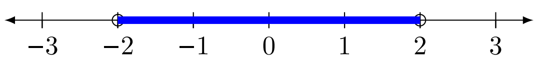
The absolute value inequality $|x| < 2$ represents the distance between x and 0 that is less than two.

You can write he above absolute as a **compound inequality** : $-2 < x < 2$.

In Interval Notation : $(-2, 2)$

In number line (graphically):

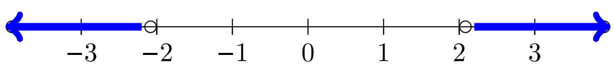


– The absolute value inequality $|x| > 2$ represents the distance between x and 0 that is greater than two.

You can write he above absolute as a **compound inequality** : $-2 > x > 2$.

In Interval Notation : $(-\infty, -2) \cup (2, \infty)$

In number line (graphically):



Inequality notation	Number line	Interval notation
$a \le x \le b$	a b	$[a, b]$
$a < x < b$	a b	(a, b)
$a \le x < b$	a b	$[a, b)$
$a < x \le b$	a b	$(a, b]$
$a \le x$	a	$[a, \infty)$
$a < x$	a	(a, ∞)
$x \le b$	b	$(-\infty, b]$
$x < b$	b	$(-\infty, b)$

Example 1 $\frac{6}{8} < \frac{z+3}{4} < \frac{18}{8}$

Solution:

As in equalities, we will get rid of the denominators first. Since 8 is the common multiple of 4 and 8 is 8, we will multiply by 8 on all sides

$8\frac{6}{8} < 8\frac{z+3}{4} < 8\frac{18}{8}$

$6 < 2(z+3) < 18$

distribute 2

$6 < 2z + 6 < 18;$

subtract 6 on both sides

$6 - 6 < 2z + 6 - 6 < 18 - 6$

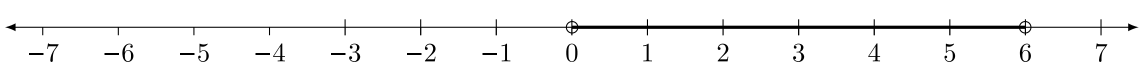
$0 < 2z < 12$

dividing all sides by 2 $\frac{0}{2} < \frac{2z}{2} < \frac{12}{2}$

$0 < z < 6$

Therefore $z \in (0, 6)$

Graphically,



Example 2 : $-27 < 5z - 7 \le 33$

Solution:

add 7 on both sides;

$-27 + 7 < 5z - 7 + 7 \le 33 + 7$

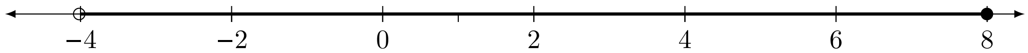
$-20 < 5z \le 40$

dividing all sides by 5

$\frac{-20}{5} < \frac{5z}{5} \le \frac{40}{5}$

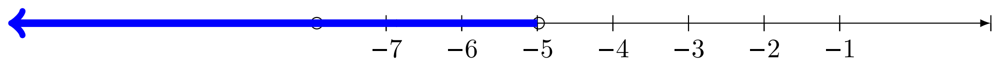
$-4 < z \le 8.$

In number line



Example 3 $9y - 1 \geq 9 + 11y$

Solution:
$9y - 9y - 1 \geq 9 + 11y - 9y$
$-1 \geq 9 + 2y$
$-1 - 9 \geq 9 - 9 + 2y$
$-10 \geq 2y$
$\frac{-10}{5} \geq \frac{2y}{2}$
$-5 \geq y$
$y \leq -5$
Therefore $y \in (-\infty, -5]$



Example 4 : $4|3 - p| > -12$
Solution:
This is a trick question. Observe that the given inequality is always true because the absolute value is always positive or zero which is greater than -12. So all real numbers are solution ie $p \in (-\infty, \infty)$.

Example 5 : $\frac{20}{4} < \frac{z+5}{2} < \frac{29}{4}$
Solution:
We first get rid of denominator. Since 4 is the common multiple of 2 and 4, we multiply all sides by 4.
$4\frac{20}{4} < 4\frac{z+5}{2} < 4\frac{29}{4}$
$20 < 2(z + 5) < 29$
$20 < 2z + 10 < 29$
$20 - 10 < 2z + 10 - 10 < 29 - 10$
$10 < 2z < 19$
$\frac{10}{2} < \frac{2z}{2} < \frac{19}{2}$
$5 < z < \frac{19}{2}$
Therefore $z \in (5, 9.5)$.

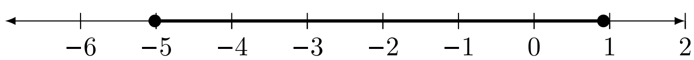
Example 6 : $|y + 2| \leq 3$ Solution:
As we did in equations, we solve two inequalities here.
$y + 2 \leq 3$
$y + 2 - 2 \leq 3 - 2$
$y \leq 1$....*

or
Remember in equalities, we simply put negative sign on the right hand side of equation and solved it. This is not the case in inequalities because multiplying by negative sign flips the inequalities sign as well. So we solve either $-(y + 2) \leq 3$ or $y + 2 \geq -3$ They are same equation.
$-(y + 2) \leq 3$
$-y - 2 \leq 3$
$-y - 2 + 2 \leq 3 + 2$
$-y \leq 5$
$y \geq -5$; multiplying by negative flips the inequality .....**

Combining * and **, we have $-5 \leq y \leq 1$

Faster Method:

We can also solve the above two cases at once using compound inequality as following:
$-3 \le y + 2 \le 3$
$-3 - 2 \le y + 2 - 2 \le 3 - 2$
$-5 \le y \le 1$



Example 7 :$3|y + 2| \ge 21$
Solution:
We start by diving both sides by 3
$\frac{3|y+2|}{3} \ge \frac{21}{3}$
$|y + 2| \ge 7$
Now we solve two inequalities;
$y + 2 \ge 7$
$y + 2 - 2 \ge 7 - 2$
$y \ge 5$.....*

or
$-(y + 2) \ge 7$
$-y - 2 \ge 7$
$-y - 2 + 2 \ge 7 + 2$
$-y \ge 9$
$y \le -9$; multiplying by negative flips the inequality,........**

Combining * and **, we have $y \in (-\infty, -9] \cup [5, \infty)$
We can also solve the above two cases at once using compound inequality as following:
$-7 \ge y + 2 \ge 7$
$-7 - 2 \ge y + 2 - 2 \ge 7 - 2$
$-9 \ge y \ge 5$