River Parishes Community College

Math 1100: College Algebra

Equations and Inequalities

2.2 Linear Equations in one Variable

Semester
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Department
Physical Science: Math

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Learning Objectives

In this section, you will learn:
♣ Distingusih the difference between Linear vs Non-linear Equations
♣ Learn to solve Linear Equations
♣ Learn the concept of Slope
♣ Given the equations of two lines, determine whether their graphs are parallel or perpendicular.
♣ Write the equation of a line parallel or perpendicular to a given line.

1 Introduction : Linear Equations, Lines and Slope

In pervious section 2.1 − *Rectangular Coordinates* we learned how co-ordinates of an order pair (x, y) lets us go from origin to any point we desire in the plane. In this section we draw a line in coordinate plane and observe that there is a very nice way to move from one point in the line to any other point in the same line using slope.

2 Linear Equation in one variable

A linear equation is an equation that can be written in the form

$$ax + b = 0$$

for a and b are real numbers (constants) and $a \neq 0$. Note that the power (exponent) of x is 1.

Examples of Linear Equation in one varibale

a. $2x - 5 = 0$
b. $2(x - 2) = 6$ can be rewritten as $2x - 10 = 0$.
c. $3x + 2 = 7x + 5$ can be rewritten as $-4x - 3 = 0$

Examples of Non-linear Equation in one varibale

a. $2x + 3y = 0$ Two Variables
b. $x^2 + 2x = 6$ The exponent (degree/power) is 2
c. $\sqrt{x} + 2 = 3$ radical (fractional) exponents.
d. $\frac{2}{x} + 3 = x + 6$ Variable in denominator

3 Solving Linear Equations in one Variable

A solution of a linear equation in one variable is a real number which, when substituted for the variable in the equation, makes the equation true.

Eg

Is 3 a solution to the equation $2x + 3 = 11$?
Original Equation: $2x + 3 = 11$
Substitute 3 for x: $2(3) + 3 = 11$
False Equation $6 + 3 \neq 11$
3 is not a solution.

Is 4 a solution to the equation $2x + 3 = 11$?
Original Equation: $2x + 3 = 11$
Substitute 4 for x: $2(4) + 3 = 11$
True Equation $8 + 3 = 11$
4 is a the solution.

Addition, Multiplication and Distributive Property of Equations

Addition

If $a = b$, then $a + c = b + c$ and $a - c = b - c$.

That is, the same number can be added to or subtracted from each side of an equation without changing the solution of the equation.

Multiplication

If $a = b$ and $c \neq 0$, then $ac = bc$ and $\frac{a}{c} = \frac{b}{c}$

That is, an equation can be multiplied or divided by the same nonzero real number without changing the solution of the equation.

Distributive

$$a(b + c) = ab + ac$$

To solve a linear equation in one variable:

1. Simplify both sides of the equation.
2. Use the addition and subtraction properties to get all variable terms
on the left-hand side and all constant terms on the right-hand side. 3. Simplify both sides of the equation.
4. Divide both sides of the equation by the coefficient of the variable.

3.1 Identity vs Inconsistent equation

While some equations does not need to have a solution, some equations always do and infinite amount. Equations which is always true for all the variables is called Identity. For example:

$$3x = 2x + x.$$

The solution set consists of all values that make the equation true. For this equation, the solution set is all real numbers because any real number substituted for x will make the equation true.

Equations which is always untrue for all variables is called inconsistent equation. For example:

$$3x = 3x + 6$$

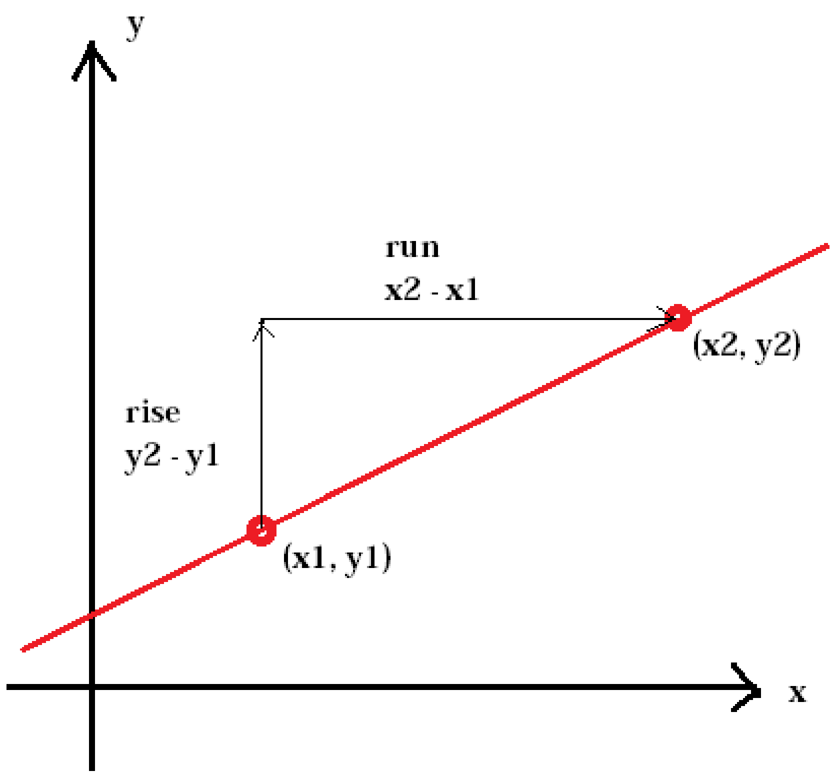
$$3x - 3x = 6$$

$$0 = 6$$

This is a false statement so there is no solutions.

4 What is Slope?

Short answer: The slope between two points is rise over run. Rise is the difference of two y values and run is difference of two x values. What this means is that we want to know how much higher the function is if you move over a certain amount to the right.
Consider the function $y = \frac{2}{1}x + 3 = 2x + 3$. By using plotting technique we learned in earlier chapter we start at the point $(0, 3)$. All points that satisfy this equation fit on the line. So if we plug in $x = 1$, then $y = 5$. By continuing this process we end up with a graphed line. The slope of this line is the distance "up," (change in y) for every distance we go "right," (change in x). So, staring at point $(0, 3)$, if we were to go 1 in the x direction (say from 0 to 1) we move up 2 in y direction and reach the point $(1, 3 + 2) = (1, 5)$. Similarly, the points $(1 + 1, 5 + 2) = (2, 7)$ and $(2 + 1, 7 + 2) = (3, 9)$ and $(3 + 1, 9 + 2) = (4, 11)$ all fit the line.



In mathematical language, the slope m of the line is

Slope Equation

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

The slope of line measures the steepness of a line. A line is either increasing, decreasing, horizontal or vertical.

Slope and Steepness

♣ A line is increasing if it goes up from left to right. The slope is positive, i.e. $m > 0$.
♣ A line is decreasing if it goes down from left to right. The slope is negative, i.e. $m < 0$.
♣ If a line is horizontal the slope is zero. This is a constant function.
♣ If a line is vertical the slope is undefined .

5 Slope-Intercept Form of a Line: $y = mx + b$

Let $(0, b)$ be the y-intercept of a line i.e it is the point where the line intersects the y axis. Let (x, y) be other any other point in the line. Now using the slope equation

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

$$m = \frac{y - b}{x - 0}$$

$$m = \frac{y - b}{x}$$

Multiplying both sides by x

$$mx = y - b$$

Adding b on both sides

$$mx + b = y$$

Besides being neat and simplified, slope-intercept form's advantage is that it gives two main features of the line it represents:
♣ **The slope is** m
♣ **The** y **intercept is** b**. i.e in order pairs :** $(0, b)$

Eg

The line $y = 2x + 1$ has slope 2 and y-intercept $(0, 1)$.

The fact that this form gives the slope and the y-intercept is the reason why it is called slope-intercept in the first place!

6 Point-Slope Form of a Line

Let (x_1, y_1) be the given point of a line. Then, let (x, y) be any other point in the given line. Using the slope formula we

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

$$m = \frac{y - y_1}{x - x_1}$$

Multiplying both sides by $(x - x_1)$

$$m(x - x_1) = y - y_1$$

This form gives the slope and a point in the line, hence this is called point-slope Form of a line.

Eg 1: Find the Equation of a line that passes through point $(1, 5)$ and has slope -2

Write the slope intercept form:

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -2(x - 1)$$

Eg 2: Find the Equation of a line that passes through points $(1, 4)$ and $(6, 19)$

Step 1: Find the slope :

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

$$m = \frac{19 - 4}{6 - 1} = \frac{15}{5} = 3.$$

Step 2: Use the point slope form:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 3(x - 1)$$

or

$$y - 19 = 3(x - 6)$$

7 Standard Form of a Line

Another way that we can represent the equation of a line is in standard form. Standard form is given as

$$Ax + By = C$$

where A, B and C are integers. The x- and y-terms are on one side of the equal sign and the constant term is on the other side. This form is nice only because it does not involve any fractions. However, to find the slope and y-intercept, it needs to be converted to other forms.

Three Forms of the Equation of Line

1. Point Slope form

$$y = mx + b.$$

2. Slope Intercept form

$$y - y_1 = m(x - x_1).$$

3. Standard Form

$$Ax + By = C.$$

Three Scenarios for finding the Equation of Line

♣ Given a slope and y-intercept , find the equation of a line.
♣ Given a slope and a point, find the equation of a line.
♣ Given two points in a line, find the equation of a line.

8 Horizontal Lines and Vertical Lines

Horizontal Line

All the points in the horizontal line have the same y-co-ordinates, so the equation of a horizontal line is given by

$$y = b.$$

The horizontal line does not rise at all, so the slope is zero .

Vertical Line

All the points in the vertical line have the same x-co-ordinates, so the equation of a horizontal line is given by

$$x = c.$$

The vertical line does run at all, so the slope is undefined because the denominator goes to zero in the slope formula .

Eg 1: Find the Equation of a line that passes through point $(1, 5)$ and $(1, -3)$

Since they have the same x co-ordiante, it is a vertical line. The slope is undefined and the equation of the line is

$$x = 1$$

.

Eg 2: Find the Equation of a line that passes through points $(1, 4)$ and $(-3, 4)$

Since they have the same y coordinate , it is a horizontal line . The slope is zero and the equation of the line is

$$y = 4.$$

9 Parallel and Perpendicular Lines

Parallel Lines

The slopes of parallel lines are always equal.

♣ Example:

♠ The equations $y = 4x + 1$ and $y = 4x + 3$ are parallel because they both have a slope of 4.

Perpendicular lines

The slopes of perpendicular lines are always opposite reciprocals of each other.

♣ Example:

- The equations $y = \frac{2}{3}x+2$ and $y = -\frac{3}{2}x+5$ are perpendicular because their slopes are negative reciprocals of each other.
- The term opposite reciprocal simply means to flip the slope and change the sign.
  - For example, if $m = \frac{4}{5}$, then a slope perpendicular to that would be $m = -\frac{5}{4}$.

Two Slopes that are perpendicular will always have product of -1

- For example:
  $\frac{4}{5} * -\frac{5}{4} = -1$

Writing Equations of Parallel and Perpendicular Lines

3 steps to write Parallel and Perpendicular Lines

1. Determine the slope of the new line.

2. Pick which equation to use:

   - Slope-intercept: $y = mx + b$.

   - Point-Slope: $y - y_1 = m(x - x_1)$.

3. Substitute values.

Example: Write the equation of a line perpendicular to $y = -\frac{1}{3}x - 5$ through the point $(-1, 4)$. Write the final answer in slope-intercept form

Step 1: **Determine the slope of the new line**.
Since it must be perpendicular to the given line, the slope of our new line must be $m = 3$.

Step 2: **Pick equation.**
Since we have the point $(-1, 4)$, we should use the point-slope equation of a line $y - y_1 = m(x - x_1)$.

Step 3: **Substitute values**.
Since $m = 3$ and $(x_1, y_1) = (-1, 4)$, we can substitute those into the point-slope equation:

$$y - y - 1 = m(x - x_1)$$

$$y - 4 = 3(x - (-1)) = 3(x + 1)$$

Now the problem says to write the final answer in slope-intercept form. To do this, we simply solve for y.

$$y - 4 = 3(x + 1)$$

$$y - 4 = 3x + 3$$

$$y = 3x + 7$$

So the line perpendicular to $y = -\frac{1}{3}x - 5$ through the point $(-1, 4)$ is $y = 3x + 7$