River Parishes Community College

Math 1100: College Algebra

Equations and Inequalities

2.1 The Rectangular Coordinate System

Semester
Sp/Fall Year

Department
Physical Science: Math

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Learning Objectives

In this section, you will learn:
♣ Plot ordered pairs in a Cartesian coordinate system.
♣ Graph equations by plotting points.
♣ Find x-intercepts and y-intercepts.
♣ Use the distance formula.
♣ Use the midpoint formula.

1   Introduction : Cartesian Co-ordinate System

Imagine you have a sheet of paper in your hand. Now draw equally spaced horizontal lines and vertically lines perpendicular to each other all over the paper. Once you fix a point where the vertical and horizontal line intersect, you can go to any other intersecting points simply by moving left/right and up/down. This is an idea of tremendous importance first thought by the Great Mathematician **Rene Descartes**. This allows us to mathematically go from one point to another point in a sheet of paper or any geometrical plane.
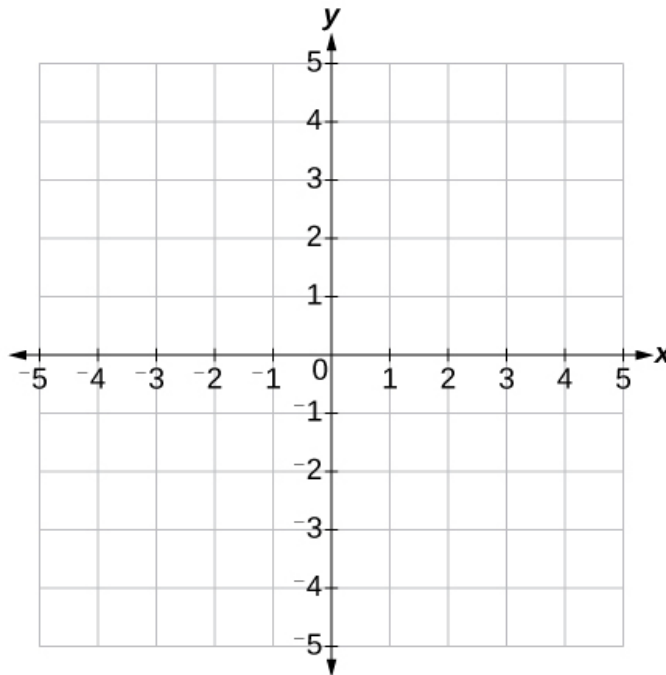
The point we fixed is called origin. The horizontal line passing through the origin is called $x - axis$. The vertical line passing through the origin is called $y - axis$. Since the plane is divided into rectangles or squares, this is called *Rectangular Coordinate System* also know as *Cartesian Coordinate System*. Each squares or rectangles are called units. Each section is called quadrants.
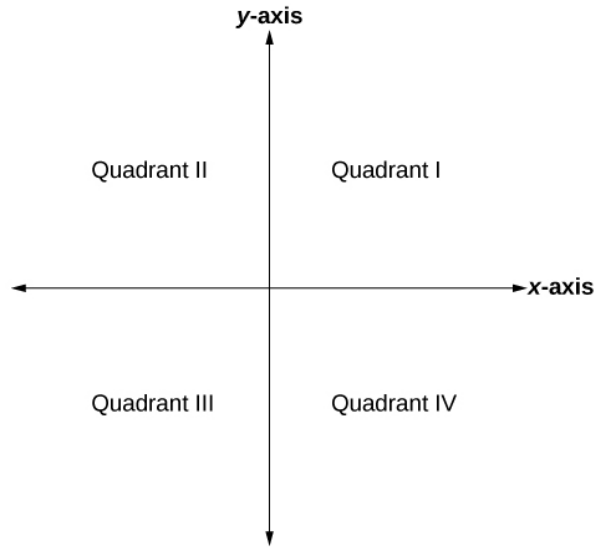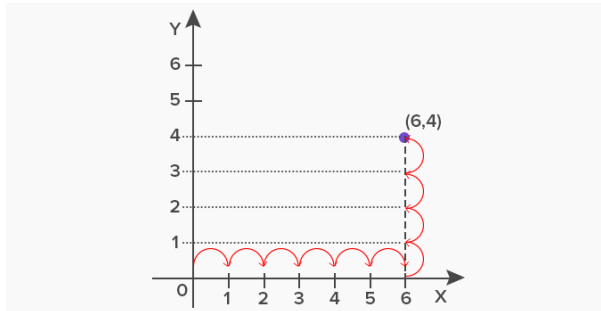

2   Order Pairs

There are several ways to reach to any point in the plane from the origin. You can first go up/down and left/right. You can also reach the same point by going left/right first , then up/down. So, by convention, we move horizontal (left or right) first , then we move vertically (up or down). This will establish that each point in the plane will be reached only one way.

Example

The point $(6, 4)$ can be reached first by moving 6 units to the right first and then 4 units up.
The point $(-6, 4)$ can be reached first by moving 6 units to the left first and then 4 units up.
The point $(6, -4)$ can be reached first by moving 6 units to the right first and then 4 units down.
The point $(-6, -4)$ can be reached first by moving 6 units to the left first and then 4 units down.

The point (x, y) is called *order pair* emphasizing that one moves x distance horizontally first, and then moves y distance vertically. x and y are called co-ordinates. So, each point in the plane is uniquely (only one way) identified by an order pair.

As the name "ordered pair" suggests, the order in which values are written in a pair is very important. The ordered pair $(6, 4)$ is different from the pair $(4, 6)$. Both represent two different points as shown below.
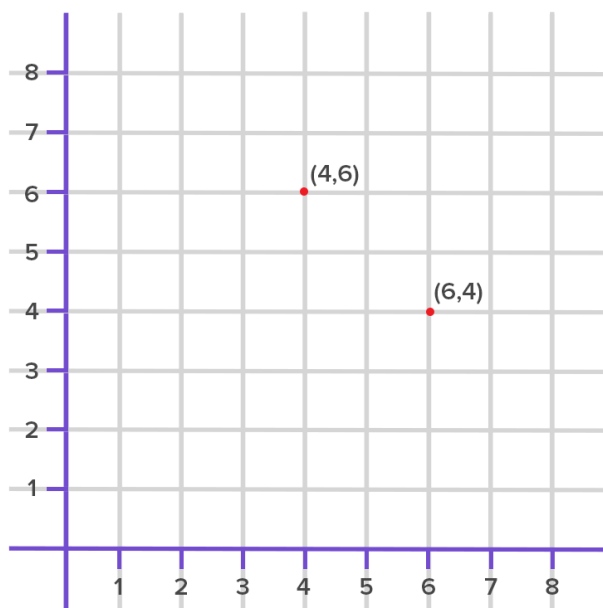
Note that when either coordinate is zero, the point must be on an axis. If the x-coordinate is zero, the point is on the y-axis. If the y-coordinate is zero, the point is on the x-axis.

3 Graphing Equations by Plotting Points

Plotting Points is one of the important way to see how the graph looks like or its behaviour around a region. Since (x, y) consists of two variables, so we can fix one and find the other one. This means we can randomly choose some values for x and solve for y or vice versa.

How to graph by plotting points given the equation ?

1. Make a table with one column labeled x, a second column labeled with the equation, and a third column listing the resulting ordered pairs.
2. Enter x-values down the first column using positive and negative values. Selecting the x-values in numerical order will make the graphing simpler.
3. Select x-values that will yield y-values with little effort, preferably ones that can be calculated mentally.
4. Plot the ordered pairs.
5. Connect the points if they form a line.

> **Example**
>
> Graph the equation $y = -x + 2$ by plotting points.
>
> | x | $y = -x + 2$ | (x, y) |
> |-----|--------------|----------|
> | -1 | $y = -(-1) + 2 = 3$ | $(-1, 3)$ |
> | 0 | $y = -(0) + 2 = 2$ | $(0, 2)$ |
> | 1 | $y = -1 + 2 = 1$ | $(1, 1)$ |
> | 2 | $y = -(2) + 2 = 0$ | $(2, 0)$ |
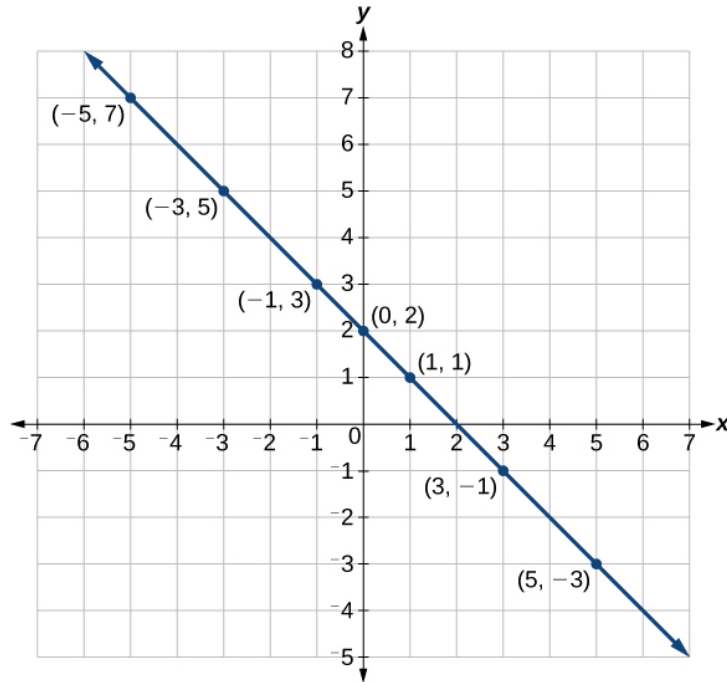
4 x-intercepts and y-intercepts

The intercepts of a graph are points at which the graph crosses the axes. The x-intercept is the point at which the graph crosses the x-axis. At this point, the y-coordinate is zero. The y-intercept is the point at which the graph crosses the y-axis. At this point, the x-coordinate is zero. To determine the x-intercept, we set y equal to zero and solve for x. Similarly, to determine the y-intercept, we set x equal to zero and solve for y.

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Eg. Find the x and y intercepts of $y = 2x + 6$

To find x intercepts set $y = 0$.
$0 = 2x + 6$
$-6 = 2x$
$-3 = x$
The graph cross the x-axis at $(-3, 0)$.
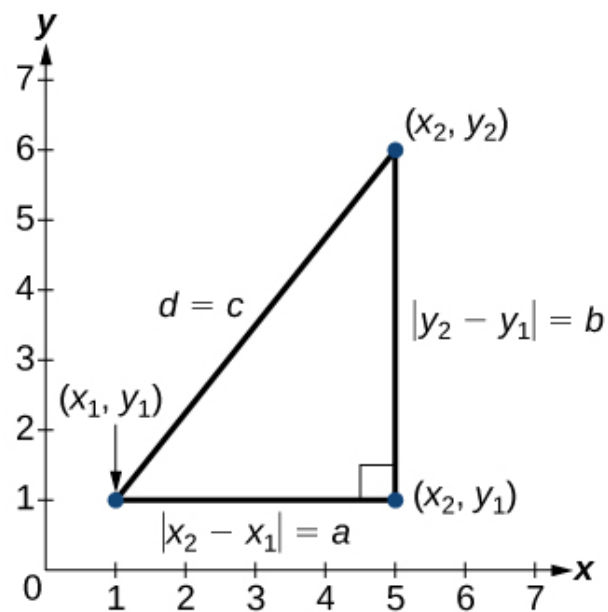
To find the y-intercept set $x = 0$.
$y = 2(0) + 6$
$y = 6$
The graph cross the y axis at $(0, 6)$.

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5 Distance Formula and Midpoint Formula

The distance formula is actually a property of a right angled triangle where $c^2 = a^2 + b^2$ as shown in the figure below:

Distance Formula

Given endpoints (x_1, y_1) and (x_2, y_2) the distance between two points is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

Given endpoints (x_1, y_1) and (x_2, y_2) the midpoint between two points is given by

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Eg. Determine the distance and Midpoint between two points $(3, -3)$ and $(-9, 2)$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-9 - 3)^2 + (2 - -3)^2}$$

$$= \sqrt{(-12)^2 + (5)^2}$$

$$= \sqrt{169}$$

$$= 13$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{3 - 9}{2}, \frac{-3 + 2}{2} \right)$$

$$= \left(\frac{-6}{2}, \frac{-1}{2} \right)$$

$$= \left(-3, \frac{-1}{2} \right)$$